



D&D Resources Ltd
Mathematics Exam Preparation Made Easy

External Achievement Standard - 91586

Practice External Assessments 3

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NCEA 3 Statistics

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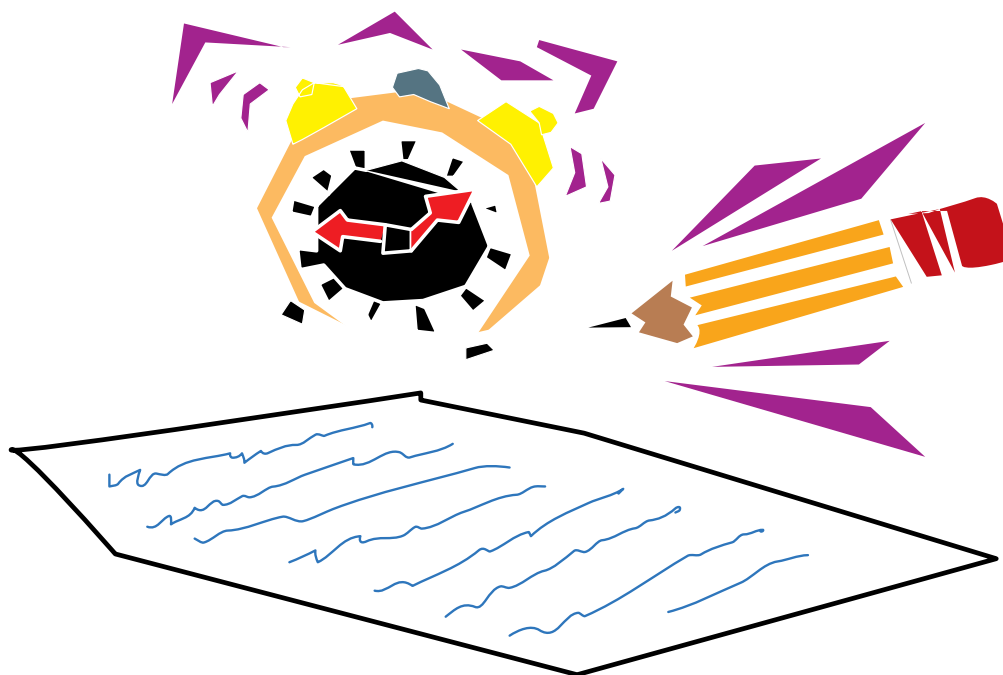
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The questions in the practice assessments are NOT in order of difficulty. Attempt all questions or you may not provide enough evidence to achieve the required standard.

Achievement Standard

91586

Apply Probability Distributions in Solving Problems

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

- ◆ This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
 - ❖ Investigate situations that involve elements of chance
 - calculating and interpreting expected values and standard deviations of discrete random variables.
 - applying distributions such as the Poisson, binomial, and normal.
- ◆ Apply probability distributions in solving problems involves:
 - ❖ selecting and using methods
 - ❖ demonstrating knowledge of concepts and terms
 - ❖ communicating using appropriate representations.
- ◆ Relational thinking involves one or more of:
 - ❖ selecting and carrying out a logical sequence of steps
 - ❖ connecting different concepts or representations
 - ❖ demonstrating understanding of concepts;
 and also relating findings to a context, or communicating thinking using appropriate statements.
- ◆ Extended abstract thinking involves one or more of:
 - ❖ devising a strategy to investigate or solve a problem
 - ❖ identifying relevant concepts in context
 - ❖ developing a chain of logical reasoning
 - ❖ making a generalisation;
 and also, where appropriate, using contextual knowledge to reflect on the answer.
- ◆ Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or statistical contexts.
- ◆ Methods include a selection from those related to:
 - ❖ discrete and continuous probability distributions
 - ❖ mean and standard deviation of random variables
 - ❖ distribution of true probabilities versus distribution of model estimates of probabilities versus distribution of experimental estimates of probabilities.

You are advised to spend 60 minutes answering this assessment.

You should show ALL working and answer ALL parts of ALL questions.

QUESTION ONE

- (a) The LTSA have identified the mean speed of vehicles travelling through a particular section of the Wellington motorway to be 108.6 km/h with a standard deviation of 9.5 km/h.

Assuming the speeds are normally distributed calculate the probability that a random vehicle, travelling through this particular section of the motorway, is travelling faster than 110 km/h.

It is estimated that 45% of the tickets issued by police officers for traffic offences are for those in the age group 18 to 25 years.

- (b) (i) If a police officer issues 8 motorists with traffic offences, calculate the probability, using an appropriate distribution, that 6 or more are aged 18 to 25 years.

- (ii) Justify your choice of probability distribution, in part i) above and identify the parameters used and state any assumptions you make.

- (c) Calculate the probability that in two successive groups of 8 motorists, which the police officer issues with traffic offences exactly 3 of each group are aged 18 to 25 years.



- (b) Elisa has an online store where customers can buy electronic products via her website.

Elisa looks at her website statistics and prints out the probability distribution graph which gives the number of sales via her website per day over a one year period (see the graph below).



- (i) Use this data to complete the table below, which shows a probability distribution model for the random variable N , the number of sales per day via Elisa's online store, and calculate the mean number of sales per day.

Give any assumption(s) that need to be made.

n	0	1	2	3	4	5	6	7
$P(N = n)$		0.15	0.20	0.25		0.1	0.05	0.025

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- (ii) Elisa has a goal to reduce the probability of zero online sales down to 0.025 in any one 24 hour period (day). How many online purchases would she have to make to achieve this?

Is this result a negative or positive for her online business?

- (c) It is expected that over the season 15 000 skiers/boarders will hire ski/board gear from the rental company located on the mountain. The amount the skiers/boarders will spend hiring gear is expected to be normally distributed with mean \$140 and standard deviation \$42.

The rental company operating on the mountain, decide to impose a fee on all rentals, in order to have enough capital to replace old hire skis, boards, jackets etc. at the end of season. The fee charged is based on the total amount spent by each skier/boarder on rental equipment (see the table below).

Rental amount (x)	$x < \$100$	$\$100 \leq x \leq \200	$x > \$200$
Fee imposed	\$5	\$9	\$15

- (i) Complete the probability distribution table below for the random variable X, the rental amount paid by skiers/boarders who hire ski/board gear from the rental company on the mountain and use it to calculate the mean and standard deviation fee imposed for a single person renting ski/board gear on the mountain.

Rental amount (x)	$x < \$100$	$\$100 \leq x \leq \200	$x > \$200$
$P(X = x)$			

- (ii) The rental company is hoping to collect at least \$125 000 from the fee to go towards the purchase of new hire gear.

By calculating the total expected amount collected, identify whether the rental company will have enough capital to replace old hire gear. Show all your working and justify your answer.

QUESTION THREE

The number of people using the coffee vending machine in the waiting room of a doctors' practice was recorded every 30 minutes over a number of weeks. The results are plotted. See the probability distribution graph below.



- (a) Which is the best model to use in order to calculate the probability of n people using the coffee vending machine within a 30 minute period? Justify your choice of distribution and calculate the parameter(s) for your distribution.

- (b) Identify one of your assumptions in part a) that may be invalid and discuss why this is the case.

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PEA 6

You are advised to spend 60 minutes answering this assessment.

You should show ALL working and answer ALL parts of ALL questions.

QUESTION ONE

- (a) In a dentist's practice it is known that a consultation with the dentist takes between 5 and 30 minutes with a usual time of 15 minutes, while a consultation with the oral hygienist takes between 15 and 45 minutes with a usual time of 35 minutes.

- (i) Identify an appropriate probability distribution model and sketch each distribution on the set of axes below.

Clearly label and include as much information as possible on the sketch for each distribution.



- (ii) Calculate the probability that two patients, one who sees the hygienist and the other the dentist, both take more than 25 minutes for their consultation.

Identify any assumption(s) you need to make.

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PEA 4

THREE				
(a)	Binomial distribution would appear to be the best model, $n = 2000$, $\pi = 0.6225$ - the number of trials (number of people in the group) is fixed. - each trial is either a skier or snow boarder (success or failure). - each trial (person) is independent of the others. - the probability of being a skier is constant, i.e. 0.6225.	identifies binomial distribution and parameters.	identifies binomial distribution and parameters AND justifies choice of distribution in context.	
(b) 1.	Binomial distribution, $n = 15$, $\pi = 0.6225$. $P(X \geq 12) = 0.1227$ (calculator)	correct probability calculated.		
2.	If the lift is providing access to the mountain then it is likely to reflect reality. If the lift is to a specific run on the mountain and because some runs are more favoured by skiers than snow boarders and vice versa it may not reflect reality.		appropriate comment on whether the result will reflect reality with justification.	
(c)	Skiers, $n = 18$, $\pi = 0.58$ $P(X \geq 10) = 0.6764$ (calculator). Snow boarders, $n = 12$, $\pi = 0.45$ $P(X \geq 7) = 0.2607$ (calculator). $P(X \geq 10 \text{ and } X \geq 7)$ $= 0.6764 \times 0.2607$ $= 0.1763$	calculates one probability correctly.	calculates two probabilities correctly.	calculates $P(X \geq 10 \text{ and } X \geq 7)$ correctly.
(d)	$\pi = 0.05$, $P(X \geq 1) > 0.7$ so $P(X = 0) = 1 - 0.7 = 0.3$ Using binomial ${}^nC_0 0.05^0 (0.95)^n < 0.3$ $0.95^n < 0.3$ $n \log 0.95 < \log 0.3$ $n > 23.47$ (24 customers).	identifies binomial distribution and makes progress towards correct answer.	identifies binomial distribution and formulates binomial inequality.	finds number of customers correctly with supporting calculations.

Marking Schedule for Questions 1, 2 and 3

N1	N2	A3	A4	M5	M6	E7	E8
One partial solution	1 of A	2 of A	3 of A	1 of M	2 of M	1 of E	2 of E